

# Significant Figures III

How significant figures relate to the experimental measurement and how to mathematically manipulate significant figures.

PART III - Handling significant figure in mathematical manipulations

## Significant Figures, an introduction:

Mathematics for manipulating significant figures (digits).

There is one rule for:

Adding and subtracting

There is another rule for:

multiplication and division

**Do not mix them up!**

There is a third rule for the “ln”, “log”, “exp” and “antilog” functions Be thankful you’re not being asked to learn it also.

## Significant Figures, an introduction:

Rule for addition and subtraction:

The absolute precision of the number resulting from an addition or subtraction is the same as the absolute precision of the least precise starting number.

Significant Figures, an introduction:

Rule for addition and subtraction:

The absolute precision of the number resulting from an addition or subtraction is the same as the absolute precision of the least precise starting number.

Rather than learning this, learn how to determine the number of significant figures by the following procedure.

## Significant Figures, an introduction:

Steps in doing addition and subtraction.

- 1) determine the absolute precision of each number. That is, determine how many to the right of the decimal separator there are. The number with the least number of digits has the highest value for the uncertainty. This highest value is the one to select.

# Significant Figures, an introduction:

Steps in doing addition and subtraction.

- 1) determine the absolute precision of each number. That is, determine how many to the right of the decimal separator there are. The number with the least number of digits has the highest value for the uncertainty. This highest value is the one to select.
- 2) add or subtract the numbers.

# Significant Figures, an introduction:

Steps in doing addition and subtraction.

- 1) determine the absolute precision of each number. That is, determine how many to the right of the decimal separator there are. The number with the least number of digits has the highest value for the uncertainty. This highest value is the one to select.
- 2) add or subtract the numbers.
- 3) round the answer to the place determined by step 1) in the answer.

# Significant Figures, an introduction:

Steps in doing addition and subtraction.

- 1) determine the absolute precision of each number. That is, determine how many to the right of the decimal separator there are. The number with the least number of digits has the highest value for the uncertainty. This highest value is the one to select.
- 2) add or subtract the numbers.
- 3) round the answer to the place determined by step 1) in the answer.

A few examples will illustrate this.

# Significant Figures, an introduction:

## Example 1:

Add the following numbers:

1.521

2.7242

8.22

# Significant Figures, an introduction:

## Example 1:

Add the following numbers:

1.521  
2.7242  
8.22

The least significant digits for each is (x being used to make this generalized):

0.00x

# Significant Figures, an introduction:

## Example 1:

Add the following numbers:

1.521

2.7242

8.22

The least significant digits for each is (x being used to make this generalized):

0.00x

0.000x

# Significant Figures, an introduction:

## Example 1:

Add the following numbers:

1.521

2.7242

8.22

The least significant digits for each is (x being used to make this generalized):

0.00x

0.000x and

0.0x

# Significant Figures, an introduction:

## Example 1:

Add the following numbers:

1.521

2.7242

8.22

The least significant digits for each is (x being used to make this generalized):

0.00x

0.000x and

0.0x

The digit is therefore 0.0x

# Significant Figures, an introduction:

## Example 1:

Add the following numbers:

1.521

2.7242

8.22

---

12.4652

but only good to the nearest 0.0x

# Significant Figures, an introduction:

## Example 1:

Add the following numbers:

1.521

2.7242

8.22

12.4652 but only good to the nearest 0.0x

Thus rounding:

Answer = 12.47

Note on rounding: If the next digit is 0 - 4 simply truncate.  
If the next digit is 5-9, truncate but add 1 to the last digit.

# Significant Figures, an introduction:

## Example 1:

Add the following numbers:

$$\begin{array}{r} 1.521 \\ 2.7242 \\ 8.22 \\ \hline 12.4652 \end{array} \quad \text{but only good to the nearest } 0.0x$$

Thus rounding:

$$\text{Answer} = 12.47$$

Note on rounding: If the next digit is 0 - 4 simply truncate. If the next digit is 5-9, truncate but add 1 to the last digit.

## Significant Figures, an introduction:

Example 2: (with same powers of 10)

Add the following numbers:

$$\begin{array}{r} 3.241 \times 10^2 \\ 8.3323 \times 10^2 \\ 1.24 \times 10^2 \\ \hline 1.28133 \times 10^3 \end{array}$$

# Significant Figures, an introduction:

Example 2: (with same powers of 10)

Add the following numbers:

$$\begin{array}{r} 3.24\mathbf{1} \times 10^2 \\ 8.332\mathbf{3} \times 10^2 \\ \mathbf{1.24} \times 10^2 \\ \hline 1.28133 \times 10^3 \end{array}$$

The diagram illustrates the addition of three numbers in scientific notation, all with a power of  $10^2$ . The numbers are  $3.241 \times 10^2$ ,  $8.3323 \times 10^2$ , and  $1.24 \times 10^2$ . The result is  $1.28133 \times 10^3$ . The digits are aligned as follows:

Number	Digit 1	Digit 2	Digit 3	Digit 4	Digit 5	Digit 6	Digit 7
$3.241 \times 10^2$	3	.	2	4	1		
$8.3323 \times 10^2$	8	.	3	3	2	3	
$1.24 \times 10^2$	1	.	2	4			
$1.28133 \times 10^3$	1	.	2	8	1	3	3

Colored circles and arcs highlight the alignment of digits and the final result:

- A red circle highlights the digit '1' in  $3.241 \times 10^2$ .
- A blue circle highlights the digit '3' in  $8.3323 \times 10^2$ .
- A grey circle highlights the digit '4' in  $1.24 \times 10^2$ .
- A red arc connects the '1' in  $3.241 \times 10^2$  to the '1' in  $1.28133 \times 10^3$ .
- A blue arc connects the '3' in  $8.3323 \times 10^2$  to the '3' in  $1.28133 \times 10^3$ .
- A grey arc connects the '4' in  $1.24 \times 10^2$  to the '4' in  $1.28133 \times 10^3$ .

## Significant Figures, an introduction:

Example 2: (with same powers of 10)

Add the following numbers:

$3.241 \times 10^2$	$0.00X \times 10^2$
$8.3323 \times 10^2$	$0.000X \times 10^2$
$1.24 \times 10^2$	$0.0X \times 10^2$
<hr/>	
$1.28133 \times 10^3$	least precise: $0.0X \times 10^2$

The number should be rounded to:

$$0.0X \times 10^2 \text{ (or } X \times 10^0)$$

Therefore the rounded number would be:

$$1.281 \times 10^3 \text{ (the "1" is } 1 \times 10^0)$$

## Significant Figures, an introduction:

Example 3: (different powers of 10)

Add the following numbers:

$$\begin{array}{r} 1.71 \times 10^{-2} \\ 8.3323 \times 10^{-3} \\ 1.24 \times 10^{-1} \\ \hline 1.494323 \times 10^{-1} \end{array}$$

## Significant Figures, an introduction:

Example 3: (different powers of 10)

Add the following numbers:

$$\begin{array}{r} 1.71 \times 10^{-2} \\ 8.3323 \times 10^{-3} \\ 1.24 \times 10^{-1} \\ \hline 1.494323 \times 10^{-1} \end{array} \quad 0.0X \times 10^{-2} \quad \equiv \quad X \times 10^{-4}$$

# Significant Figures, an introduction:

Example 3: (different powers of 10)

Add the following numbers:

$$\begin{array}{r} 1.71 \times 10^{-2} \\ 8.3323 \times 10^{-3} \\ 1.24 \times 10^{-1} \\ \hline 1.494323 \times 10^{-1} \end{array}$$
$$\begin{array}{r} 0.0X \times 10^{-2} \\ 0.000X \times 10^{-3} \\ \equiv X \times 10^{-4} \\ \equiv X \times 10^{-7} \end{array}$$

# Significant Figures, an introduction:

Example 3: (different powers of 10)

Add the following numbers:

$$\begin{array}{r} 1.71 \times 10^{-2} \\ 8.3323 \times 10^{-3} \\ 1.24 \times 10^{-1} \\ \hline 1.494323 \times 10^{-1} \end{array} \quad \begin{array}{r} 0.0X \times 10^{-2} \\ 0.000X \times 10^{-3} \\ 0.0X \times 10^{-1} \end{array} \quad \begin{array}{r} \equiv X \times 10^{-4} \\ \equiv X \times 10^{-7} \\ \equiv X \times 10^{-3} \end{array}$$

# Significant Figures, an introduction:

Example 3: (different powers of 10)

Add the following numbers:

$$\begin{array}{r} 1.71 \times 10^{-2} \\ 8.3323 \times 10^{-3} \\ \underline{1.24 \times 10^{-1}} \\ 1.494323 \times 10^{-1} \end{array} \quad \begin{array}{r} 0.0X \times 10^{-2} \\ 0.000X \times 10^{-3} \\ 0.0X \times 10^{-1} \end{array} \quad \begin{array}{l} \equiv X \times 10^{-4} \\ \equiv X \times 10^{-7} \\ \equiv X \times 10^{-3} \end{array}$$

least precise:  
 $X \times 10^{-3}$

The number should be rounded to:

$X \times 10^{-3}$ . Therefore the rounded number should be:  $1.49 \times 10^{-1}$

# Significant Figures, an introduction:

## Example 3: (different powers of 10)

Add the following numbers:

$$\begin{array}{r} 1.71 \times 10^{-2} \\ 8.3323 \times 10^{-3} \\ 1.24 \times 10^{-1} \\ \hline 1.494323 \times 10^{-1} \end{array}$$

$$\begin{array}{l} 0.0X \times 10^{-2} \quad \equiv \quad X \times 10^{-4} \\ 0.000X \times 10^{-3} \quad \text{or:} \quad X \times 10^{-7} \\ 0.0X \times 10^{-1} \quad \quad \quad X \times 10^{-3} \\ \text{least precise:} \\ X \times 10^{-3} \end{array}$$

The number should be rounded to:

$X \times 10^{-3}$ . Therefore the rounded number should be:  $1.49 \times 10^{-1}$

## Significant Figures, an introduction:

Rule for multiplication and division:

The relative precision of the number resulting from a multiplication or division is the same as the relative precision of the starting number that has the highest value for the relative precision (i.e. the relatively least precise number).

## Significant Figures, an introduction:

### Rule for multiplication and division:

The relative precision of the number resulting from a multiplication or division is the same as the relative precision of the starting number that has the highest value for the relative precision (i.e. the relatively least precise number).

Again it is advised to learn the procedure rather than the statement.

## Significant Figures, an introduction:

Steps in doing multiplication and division:

- 1) Determine the number of significant figures in each number to be multiplied or involved in division.

## Significant Figures, an introduction:

### Steps in doing multiplication and division:

1) Determine the number of significant figures in each number to be multiplied or involved in division.

2) The answer should have the same number of digits as the least number of digits determined in step 1).

A few examples will illustrate this.

## Significant Figures, an introduction:

Example 1:

multiply the following numbers:

$$\begin{array}{r} 1.54 \\ \times 2.3435 \\ \times 7.222 \\ \hline 26.0641 \end{array}$$

# Significant Figures, an introduction:

Example 1:

multiply the following numbers:

$$\begin{array}{r} 1.54 \\ \times 2.3435 \\ \times \underline{7.222} \\ \hline 26.0641 \end{array}$$

3 sig. figs.

5 sig. figs.

4 sig. figs.

## Significant Figures, an introduction:

Example 1:

multiply the following numbers:

1.54	3 sig. figs.
x 2.3435	5 sig. figs.
x <u>7.222</u>	4 sig. figs.
26.0641 . .	should therefore have 3 sig. figs

round to 3 sig. figs.:

Answer = 26.1

## Significant Figures, an introduction:

Example 2: Multiply the following numbers:

$$\begin{array}{r} 7.354 \times 10^{-2} \\ \times 1.24535 \times 10^{-6} \\ \times 2.2 \times 10^3 \\ \hline 2.0148 \dots \times 10^{-4} \end{array}$$

## Significant Figures, an introduction:

Example 2: Multiply the following numbers:

$$\begin{array}{r} 7.354 \times 10^{-2} \\ \times 1.24535 \times 10^{-6} \\ \times 2.2 \times 10^3 \\ \hline 2.0148 \dots \times 10^{-4} \end{array}$$

round to 2 sig. figs.:  
Answer =  $2.0 \times 10^{-4}$

4 sig. figs.

6 sig. figs.

2 sig. figs.

need 2 sig. figs

## Significant Figures, an introduction:

Example 2: Multiply the following numbers:

$$\begin{array}{r} 7.354 \times 10^{-2} \\ \times 1.24535 \times 10^{-6} \\ \times 2.2 \times 10^3 \\ \hline 2.0148 \dots \times 10^{-4} \end{array}$$

4 sig. figs.

6 sig. figs.

2 sig. figs.

need 2 sig. figs

round to 2 sig. figs.:

$$\text{Answer} = 2.0 \times 10^{-4}$$

Note the power of 10 is much easier to handle here. **But do not be lazy. DO NOT use this rule for addition and subtraction!!!!**

# **Significant Figures III**

How significant figures relate to the experimental measurement and how to mathematically manipulate significant figures.

THE END of PART III