Significant Figures

How significant figures relate to the experimental measurement and how to mathematically manipulate significant figures.

An introduction:

What are significant figures (digits)?

In every-day expression, such as in the newspaper, when one come across a number such as:

$12,000,000

(more or less)

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In every-day expression, such as in the newspaper, when one come across a number such as:

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one does not normally take the last 6 “0"s as literally meaning 12 million dollars to the very last dollar.

An introduction:

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Likewise, when one see a measurement such as 4", one usually assumes that the value is good to at least 1/8", otherwise one would have written it so.

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What are significant figures (digits)?

In both of these cases, the numbers would be written differently in scientific writing.

For example:

An introduction:

What are significant figures (digits)?

Let's assume in the first case that the 12 million dollars was known to the nearest hundred-thousand dollars. In scientific writing then this would be written:

$\$1.20 \times 10^{7}$

where the last "0" indicates the quantity is known to the 1×10^{5} dollar.

An introduction:

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The second case is a bit problematic, since “English” units are not normally recorded decimally. Let’s, however, assume that it was known to the nearest hundredth inch. Then the value should be written:

4.00 in

with the last “0” indicating this.

An introduction:

What are significant figures (digits)?

Thus, all the digits in scientific writing have meaning, unlike the popular method of writing numbers.

First a few definitions:

“accuracy” - how close a reading is to the real value or how closely an instrument can read a measurement to its true value.

“precision” - how reproducibly one can perform a measurement or how closely one can read an instrument.

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Reading = 1.1 m
Actually = 1.2 m

Difference = 0.1 m error in accuracy

First a few definitions:

“precision” - how reproducibly one can perform a measurement or how closely one can read an instrument.

Example: 3 measurements:

1.01 m
1.02 m
1.00 m

precision is about ± 0.01 m

First a few definitions:

“precision” - how reproducibly one can perform a measurement or how closely one can read an instrument.

“high precision” means the number is ± a low value, low uncertainty in the measurement.

“low precision” means the number is ± a high value, high uncertainty in the measurement.

First a few definitions:

“absolute precision” the absolute value in the units of the measurement of the uncertainty of the measurement.

“relative precision” the absolute precision divided by the measurement itself to yield a dimensionless quantity. Sometimes it is expressed as a percent.

First a few definitions:

"least significant figure (or digit)" the digit in a number that is written the furthest to the right in the number.

for: 1.20<u>3</u> it is the "3"
for: 5.00<u>0</u> it is the last "0"
for: $1.\underline{2} \times 10^3$ it is the "2" (or 2×10^2)
for: $5.7\underline{0} \times 10^{-4}$ it is the "0" (or 0×10^{-6})

(underlines not normally given)

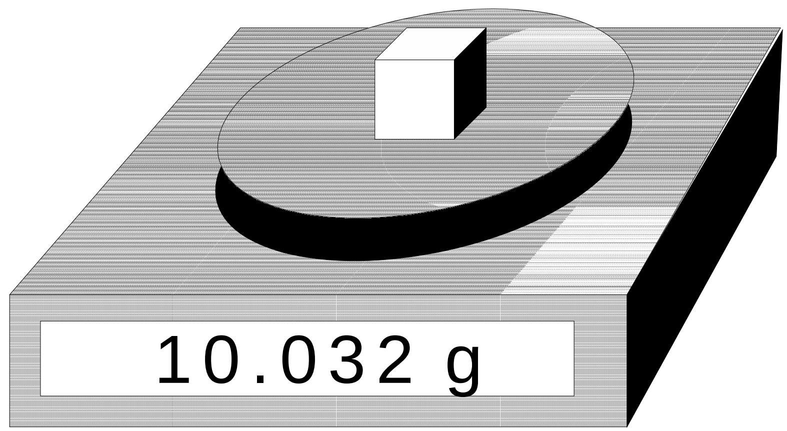
How does one determine the significant figures?

Significant figures arise out of the measurement that one makes. The normal rules are as follows:

For digital devices, read all the digits. If a digit is unstable, estimate it as the last digit.

For analogue devices, read one more digit than the normal demarcation.

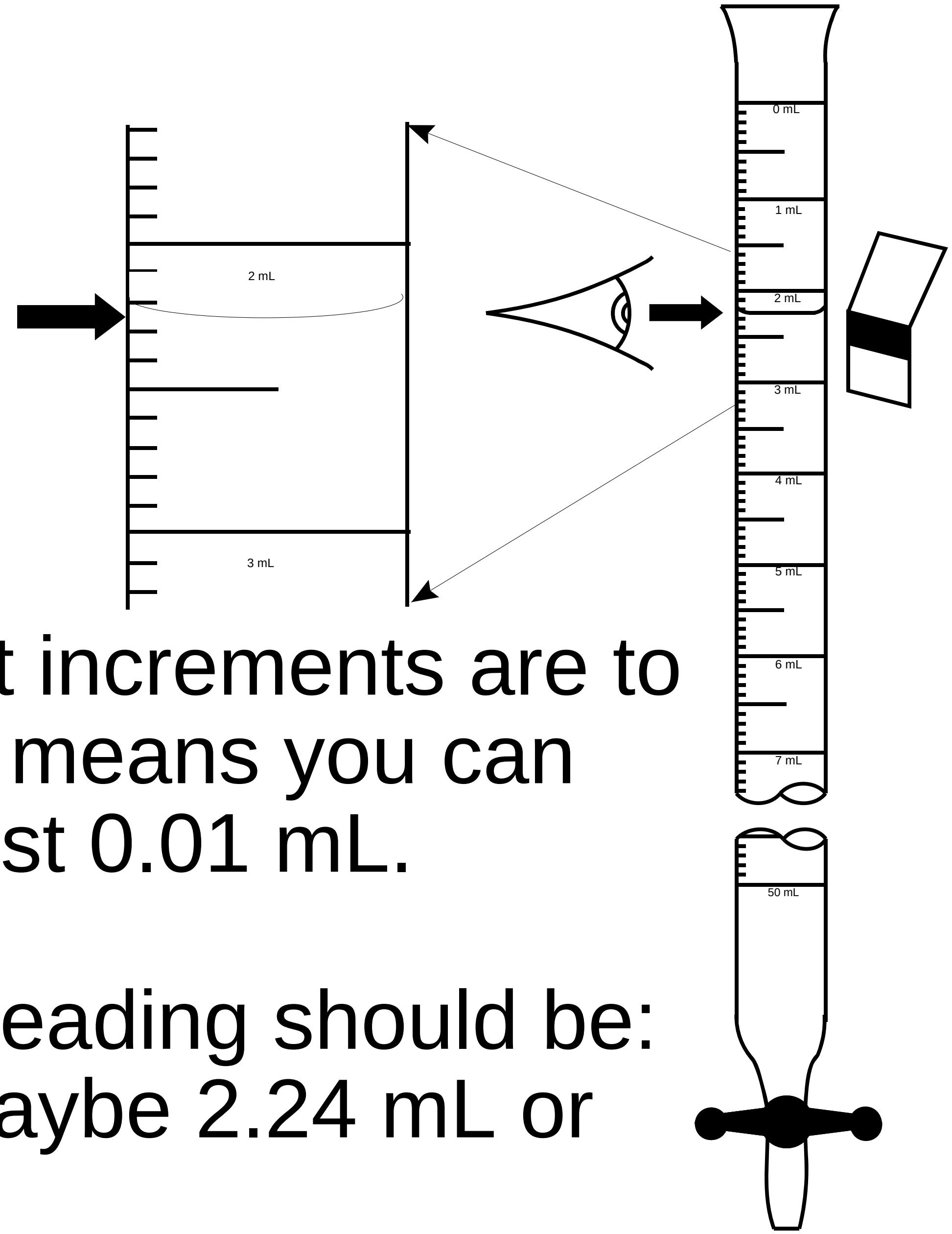
Reading a digital device should be simple. Simply record <u>all</u> the digits (always).



The above represents a digital balance. The reading is 10.032 g. The last digit, the “2”, is the least significant digit and must be recorded to indicate a sensitivity of ±0.001 g.

An example of an analogue device:

The burette:



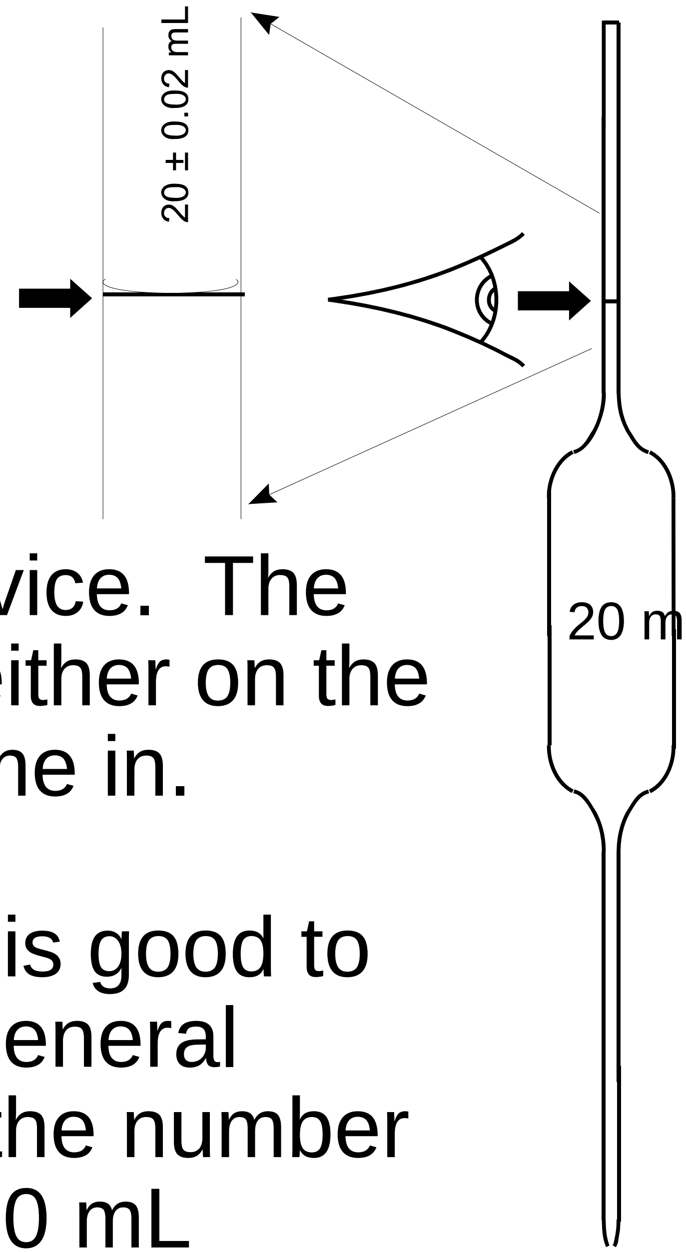
Here the smallest increments are to the 0.1 mL. This means you can read to the nearest 0.01 mL.

In this case, the reading should be:

2.25 mL (or maybe 2.24 mL or 2.26 mL)

Analogue device example 2:

the pipet:



This is a one reading device. The precision of it is written either on the pipet or on the box it came in.

In this case, the reading is good to ±0.02 mL. (typical of a General Chemistry pipet). Thus the number should be written: 20.00 mL

Mathematics for manipulating significant figures (digits).

There is one rule for:

Adding and subtracting

There is another rule for:

multiplication and division

Do not mix them up!

There is a third rule for the “ln” and “exp” functions. Be thankful you’re not being asked to learn it also.

Rule for addition and subtraction:

The absolute precision of the number resulting from an addition or subtraction is the same as the absolute precision of the starting number that has the highest value for the absolute precision (i.e. the least precise number).

Steps in doing addition and subtraction.

1) determine the absolute precision of each number. That is, determine how many places to the left of the decimal separator is the least significant digit of each number. The digit with the highest value is the one to select.

2) add or subtract the numbers.

3) round the answer to the place determined by step 1) in the answer.

A few examples will illustrate this.

Example 1:

Add the following numbers:

1.521
2.7242
8.22

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1.521
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8.22

The least significant digits for each is (x being used to make this generalized):

0.00x
0.000x and
0.0x

The digit is therefore 0.0x

Example 1:

Add the following numbers:

```
   1.521
   2.7242
   8.22
 ---------
 12.4652    but only good to the nearest 0.0x
```

Thus rounding:

Answer = 12.47

Note on rounding: If the next digit is 0 - 4 simply truncate.
If the next digit is 5-9, truncate but add 1 to the last digit.

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Add the following numbers:

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\begin{array}{r}
1.521 \\
2.7242 \\
8.22 \\
\hline
12.4652
\end{array}
$$

but only good to the nearest 0.0x

Thus rounding:

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If the next digit is 5-9, truncate but add 1 to the last digit.

Example 2: (with same powers of 10)

Add the following numbers:

$$\begin{array}{l} 3.241 \times 10^2 \\ 8.3323 \times 10^2 \\ \underline{1.24 \times 10^2 \qquad\quad} \\ 1.28133 \times 10^3 \end{array}$$

Example 2: (with same powers of 10)

Add the following numbers:

3.241×10^2	$0.00X \times 10^2$
8.3323×10^2	$0.000X \times 10^2$
$\underline{1.24 \times 10^2}$	$0.0X \times 10^2$
1.28133×10^3	least precise:
	$0.0X \times 10^2$

The number should be rounded to:

$0.0X \times 10^2$ (or $X \times 10^0$)

Therefore the rounded number would be:

$1.28\underline{1} \times 10^3$ (the "1" is 1×10^0)

Example 3: (different powers of 10)

Add the following numbers:

$$
\begin{array}{l}
1.71 \times 10^{-2} \\
8.3323 \times 10^{-3} \\
\underline{1.24 \times 10^{-1}} \\
1.494323 \times 10^{-1}
\end{array}
$$

Example 3: (different powers of 10)

Add the following numbers:

1.71×10^{-2}	$0.0X \times 10^{-2}$	$\equiv$	$X \times 10^{-4}$
8.3323×10^{-3}	$0.000X \times 10^{-3}$	or:	$X \times 10^{-7}$
1.24×10^{-1}	$0.0X \times 10^{-1}$		$X \times 10^{-3}$
1.494323×10^{-1}			least precise: $X \times 10^{-3}$

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1.494323×10^{-1}			least precise: $X \times 10^{-3}$

The number should be rounded to:

$X \times 10^{-3}$

Therefore the rounded number would be:

1.49×10^{-1}

Rule for multiplication and division:

The relative precision of the number resulting from a multiplication or division is the same as the relative precision of the starting number that has the highest value for the relative precision (i.e. the relatively least precise number).

Steps in doing multiplication and division:

1) Determine the number of significant figures in each number to be multiplied or involved in division.

2) The answer should have the same number of digits as the least number of digits determined in step 1).

A few examples will illustrate this.

Example 1:

multiply the following numbers:

```
    1.54
x  2.3435
x  7.222
---------
26.0641
```

Example 1:

multiply the following numbers:

	1.54	3 sig. figs.
x	2.3435	5 sig. figs.
x	7.222	4 sig. figs.

26.0641 . . should therefore have 3 sig. figs

round to 3 sig. figs.:

Answer = 26.1

Example 2:

multiply the following numbers:

$$
\begin{array}{ll}
 & 7.354 \times 10^{-2} \\
\times & 1.24535 \times 10^{-6} \\
\times & 2.2 \times 10^{3} \\
\hline
 & 2.0148\ldots \times 10^{-4}
\end{array}
$$

Example 2:

multiply the following numbers:

	7.354×10^{-2}	4 sig. figs.
x	1.24535×10^{-6}	6 sig. figs.
x	2.2×10^{3}	2 sig. figs.
	$2.0148 \ldots \times 10^{-4}$	need 2 sig. figs

round to 2 sig. figs.:

Answer = 2.0×10^{-4}

Note the power of 10 is much easier to handle here.

The End